MULTIPLE FRAME ESTIMATION WITH STRATIFIED OVERLAP DOMAIN

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The multiple frame estimator developed by Hartley [1] is extended to take advantage of stratification within the overlap domain for two frame estimation. The overlap domain strata are defined by a stratified list sampling frame. Matching area frame units against list units enables each stratum to be estimated by both frames. A vector of optimum weights combines the stratum estimates from the two frames. The variance of the proposed estimator is shown to be equal to or smaller than the variance of the Hartley estimator. An example using survey data compares the estimate and variance of the proposed estimator to those of other estimators not utilizing stratification.

1. INTRODUCTION

The multiple frame procedure is a common tool of the survey statistician. To estimate a specified variable, random samples are drawn from two or more sampling frames. Most multiple frame surveys involve only two frames:

1. an area frame where the sample unit is a segment of land.

2. a list frame where the sample unit is a name on a list.

Each frame has its own advantages. The list frame is often easier and cheaper to apply. Therefore, *for the same cost*, an estimate using only the list frame generally has a smaller sampling error than an estimate using only the area frame. However, the list frame rarely covers the whole population while the area frame usually does cover the whole population.

The subpopulation which is covered by the area frame but not the list frame is called the "nonoverlap" domain. The remaining part of the population, which is covered by both frames, is called the "overlap" domain. By combining the two estimates of the "overlap" domain, a single multiple frame estimator may be obtained using both the area and list frames. From work done by Hartley [1], the multiple frame estimator of a total, $\hat{Y}$, may be expressed by:

$$(1) \quad \hat{Y}_H = \hat{Y}_{nol} + q\hat{Y}_a + p\hat{Y}_\ell$$

where $\hat{Y}_{nol}$ = the area frame estimate of the "nonoverlap" domain

$\hat{Y}_a$ = the area frame estimate of the "overlap" domain

$\hat{Y}_\ell$ = the list frame estimate of the "overlap" domain

q = weight attached to the area frame estimate of the "overlap" domain

p = weight attached to the list frame estimate of the "overlap" domain

$p+q=1$.

The variance of $\hat{Y}_H$ is:

$$(2) \quad Var(\hat{Y}_H) = Var(\hat{Y}_{nol}) + q^2Var(\hat{Y}_a) + p^2Var(\hat{Y}_\ell) + 2qCov(\hat{Y}_{nol},\hat{Y}_a)$$

where $Var(\cdot)$ denotes a variance and $Cov(\cdot,\cdot)$ denotes a covariance.

2. ALTERNATIVE EXPRESSIONS OF BASIC ESTIMATOR

One widely used multiple frame estimator is the "screening" estimator. The "screening" estimator is equation (1) with $p = 1$ and $q = 0$, i.e.:

$$\hat{Y}_{Screen} = \hat{Y}_{nol} + (0)\hat{Y}_a + (1)\hat{Y}_\ell = \hat{Y}_{nol} + \hat{Y}_\ell.$$

Obviously, the variance of $\hat{Y}_{Screen}$ is:

$$Var(\hat{Y}_{Screen}) = Var(\hat{Y}_{nol}) + Var(\hat{Y}_\ell)$$

because an area frame estimate is independent of a list frame estimate.

The area frame estimate of the total of the entire population may also be expressed in terms of equation (1) with $p = 0$ and $q = 1$, i.e.:

$$\hat{Y}_{Area} = \hat{Y}_{nol} + (1)\hat{Y}_a + (0)\hat{Y}_\ell = \hat{Y}_{nol} + \hat{Y}_a.$$

The variance of $\hat{Y}_{Area}$ is:

$$Var(\hat{Y}_{Area}) = Var(\hat{Y}_{nol}) + Var(\hat{Y}_a) + 2Cov(\hat{Y}_{nol},\hat{Y}_a).$$

Since $p+q=1$, an alternative expression for Hartley's estimator is:

$$\begin{aligned}\hat{Y}_H &= \hat{Y}_{nol} + (1-p)\hat{Y}_a + p\hat{Y}_\ell \\ &= \hat{Y}_{nol} + \hat{Y}_a + p(\hat{Y}_\ell-\hat{Y}_a) \\ &= \hat{Y}_{Area} + p(\hat{Y}_\ell-\hat{Y}_a)\end{aligned}$$

Similarly, the variance of $\hat{Y}_H$ may be written as:

$$(4) \quad Var(\hat{Y}_H) = Var(\hat{Y}_{Area}) - 2pCov(\hat{Y}_{Area},\hat{Y}_a) + p^2[Var(\hat{Y}_\ell) + Var(\hat{Y}_a)]$$

If one has the data to compute $\hat{Y}_a$, the "screening" estimator appears inefficient because it wastes this information. Better use may be made of data from both frames by combining $\hat{Y}_a$ and $\hat{Y}_\ell$ using an optimum p and q based on the minimization of the variance of $\hat{Y}_H$. From equation (4) optimum p is obtained as follows:

$$\frac{\partial Var(\hat{Y})}{\partial p} = -2pCov(\hat{Y}_{Area},\hat{Y}_a)+2p[Var(\hat{Y}_\ell)+Var(\hat{Y}_a)]$$

Setting this equal to zero and solving for p gives:

$$(5) \quad p_{opt} = \frac{Cov(\hat{Y}_{Area},\hat{Y}_a)}{Var(\hat{Y}_a)+Var(\hat{Y}_\ell)}$$

or $$p_{opt} = \frac{Var(\hat{y}_a)+Cov(\hat{y}_{no\ell},\hat{y}_a)}{Var(\hat{y}_a)+Var(\hat{y}_\ell)}$$

Thus, Hartley's estimator, $\hat{y}_H$, is:

$$\hat{y}_H = \hat{y}_{Area} + p_{opt}\,(\hat{y}_\ell-\hat{y}_a)$$

and the variance in terms of p_{opt} is derived by substituting equation (5) into equation (4) so that:

$$(6)\quad Var(\hat{y}_H)=Var(\hat{y}_{Area})-2\,\frac{[Cov(\hat{y}_{Area},\hat{y}_a)]^2}{Var(\hat{y}_a)+Var(\hat{y}_\ell)}$$

$$+\left[\frac{Cov(\hat{y}_{Area},\hat{y}_a)}{Var(\hat{y}_a)+Var(\hat{y}_\ell)}\right]^2 [Var(\hat{y}_a)+Var(\hat{y}_\ell)]$$

$$=Var(\hat{y}_{Area}) - \frac{[Cov(\hat{y}_{Area},\hat{y}_a)]^2}{[Var(\hat{y}_a)+Var(\hat{y}_\ell)]}$$

$$=Var(\hat{y}_{Area})-p_{opt}\,Cov(\hat{y}_{Area},\hat{y}_a).$$

3. PROPOSED STRATUM MULTIPLE FRAME ESTIMATOR

The purpose of this paper is to extend the Hartley estimator to the case where the list frame is stratified. In this case:

$$\hat{y}_\ell = \hat{y}_{\ell(1)} + \hat{y}_{\ell(2)} + \ldots + \hat{y}_{\ell(k)}$$

where $\hat{y}_{\ell(h)}$ = list frame estimate of the total in the h^{th} stratum of the list, and there are k list strata. However, just as the area frame sample can be classified by domain to make an "overlap" estimate, $\hat{y}_a$, the area frame sample can also be matched with the corresponding list unit within the overlap domain to provide an "overlap" estimate for each stratum on the list:

$$\hat{y}_a = \hat{y}_{a(1)} + \hat{y}_{a(2)} + \ldots + \hat{y}_{a(k)}.$$

Using this list stratification yields an estimate of the form:

$$\hat{y}_{Strata}=\hat{y}_{no\ell} + \sum_{h=1}^{k} q_h\,\hat{y}_{a(h)} + \sum_{h=1}^{k} p_h\,\hat{y}_{\ell(h)},$$

where $q_h + p_h = 1$ for $h=1, 2, \ldots, k$. The proposed estimator may also be written:

$$\hat{y}_{Strata}=\hat{y}_{no\ell} + \sum_{h=1}^{k} (1-p_h)\hat{y}_{a(h)} + \sum_{h=1}^{k} p_h\,\hat{y}_{\ell(h)}$$

$$=\hat{y}_{no\ell} + \sum_{h=1}^{k} \hat{y}_{a(h)} + \sum_{h=1}^{k} p_h\,[\hat{y}_{\ell(h)}-\hat{y}_{a(h)}]$$

$$=\hat{y}_{Area} + \sum_{h=1}^{k} p_h\,[\hat{y}_{\ell(h)}-\hat{y}_{a(h)}].$$

It is easier for the following work to change to matrix notation.

Let:

$$\underline{p} = \begin{bmatrix} p_1 \\ p_2 \\ \cdot \\ \cdot \\ \cdot \\ p_k \end{bmatrix}$$

$$\underline{\hat{y}}_\ell = \begin{bmatrix} \hat{y}_{\ell(1)} \\ \hat{y}_{\ell(2)} \\ \cdot \\ \cdot \\ \cdot \\ \hat{y}_{\ell(k)} \end{bmatrix} \qquad \underline{\hat{y}}_a = \begin{bmatrix} \hat{y}_{a(1)} \\ \hat{y}_{a(2)} \\ \cdot \\ \cdot \\ \cdot \\ \hat{y}_{a(k)} \end{bmatrix}$$

Then:

$$(7)\quad \hat{y}_{Strata} = \hat{y}_{Area} + \underline{p}'\,[\underline{\hat{y}}_\ell - \underline{\hat{y}}_a].$$

The variance of $\hat{y}_{Strata}$ is:

$$Var(\hat{y}_{Strata}) = Var(\hat{y}_{Area}) + \underline{p}'\,[\Sigma_\ell+\Sigma_a]\underline{p} + 2Cov(\hat{y}_{Area},\,\underline{p}'\,[\underline{\hat{y}}_\ell - \underline{\hat{y}}_a])$$

$$= Var(\hat{y}_{Area}) + \underline{p}'\,[\Sigma_\ell+\Sigma_a]\,\underline{p} - 2\underline{p}'\,Cov(\hat{y}_{Area},\underline{\hat{y}}_a)$$

where:

Σ_ℓ = variance-covariance matrix of $\underline{\hat{y}}_\ell$

$$\Sigma_\ell = \begin{bmatrix} Var(\hat{y}_{\ell(1)}) & 0 & \cdots & 0 \\ 0 & Var(\hat{y}_{\ell(2)}) & \cdots & 0 \\ \cdot & \cdot & \ddots & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 0 & 0 & & Var(\hat{y}_{\ell(k)}) \end{bmatrix}$$

Σ_a = variance-covariance matrix of $\underline{\hat{y}}_a$

$$\Sigma_a = \begin{bmatrix} Var(\hat{y}_{a(1)}) & Cov(\hat{y}_{a(1)},\hat{y}_{a(2)}) & \cdots & Cov(\hat{y}_{a(1)},\hat{y}_{a(k)}) \\ Cov(\hat{y}_{a(2)},\hat{y}_{a(1)}) & Var(\hat{y}_{a(2)}) & \cdots & Cov(\hat{y}_{a(2)},\hat{y}_{a(k)}) \\ \vdots & \vdots & \ddots & \vdots \\ Cov(\hat{y}_{a(k)},\hat{y}_{a(1)}) & Cov(\hat{y}_{a(k)},\hat{y}_{a(2)}) & \cdots & Var(\hat{y}_{a(k)}) \end{bmatrix}$$

and $$Cov(\hat{y}_{Area},\underline{\hat{y}}_a) = \begin{bmatrix} Cov(\hat{y}_{Area},\hat{y}_{a(1)}) \\ Cov(\hat{y}_{Area},\hat{y}_{a(2)}) \\ \vdots \\ Cov(\hat{y}_{Area},\hat{y}_{a(k)}) \end{bmatrix}$$

We will require that Σ_ℓ and Σ_a be positive definite.

To find $\underline{p}$ we use the criterion of minimizing the variance of $\hat{y}_{Strata}$. Therefore, we set:

$$\frac{\partial Var(\hat{y}_{Strata})}{\partial \underline{p}} = \underline{0}$$

and solve for the optimum value of $\underline{p}$. Then

$$\frac{\partial Var(y_{Strata})}{\partial \underline{p}} = 0$$

yields:

$$2(\Sigma_\ell+\Sigma_a)\underline{p} - 2Cov(\hat{y}_{Area},\underline{\hat{y}}_a) = 0$$

$$(\Sigma_\ell+\Sigma_a)\underline{p} = Cov(\hat{y}_{Area},\underline{\hat{y}}_a).$$

So, the optimum value of p is:

$$\underline{p}_{opt} = (\Sigma_\ell+\Sigma_a)^{-1}\ Cov(\hat{y}_{Area},\underline{\hat{y}}_a).$$

Checking the second derivative verifies $\underline{p}_{opt}$ actually yields the <u>minimum</u> extremum of $Var(\hat{y}_{Strata})$.

4. COMPARING VARIANCES

Replace $\underline{p}$ with $\underline{p}_{opt} = (\Sigma_\ell+\Sigma_a)^{-1}Cov(\hat{y}_{Area},\underline{\hat{y}}_a)$ to yield:

$$(8)\quad Var(\hat{y}_{Strata}) = Var(\hat{y}_{Area}) - Cov(\hat{y}_{Area},\underline{\hat{y}}_a)'[\Sigma_\ell+\Sigma_a]^{-1}Cov(\hat{y}_{Area},\hat{y}_a),$$

or

$$Var(\hat{y}_{Strata}) = Var(\hat{y}_{Area}) - \underline{p}'Cov(\hat{y}_{Area},\underline{\hat{y}}_a).$$

Since Σ_ℓ and Σ_a are positive definite, and thus $\Sigma_\ell + \Sigma_a$ and $(\Sigma_\ell+\Sigma_a)^{-1}$ are positive definite:

$$Cov(\hat{y}_{Area},\underline{\hat{y}}_a)'[\Sigma_\ell+\Sigma_a]^{-1}Cov(\hat{y}_{Area},\underline{\hat{y}}_a) > 0.$$

Therefore, equation (8) shows:

$$Var(\hat{y}_{Strata}) < Var(\hat{y}_{Area}).$$

It is also possible to show that:

$$Var(\hat{y}_{Strata}) \leq Var(\hat{y}_H).$$

Remember:

$$Var(\hat{y}_H) = Var(\hat{y}_{Area}) - \frac{[Cov(\hat{y}_{Area},\hat{y}_a)]^2}{[Var(\hat{y}_a)+Var(\hat{y}_\ell)]}$$

and:

$$Var(\hat{y}_S) = Var(\hat{y}_{Area}) - Cov(\hat{y}_{Area},\underline{\hat{y}}_a)'[\Sigma_a+\Sigma_\ell]^{-1}\ Cov(\hat{y}_{Area},\underline{\hat{y}}_a).$$

Therefore, $Var(\hat{y}_{Strata}) \leq Var(\hat{y}_H)$ if:

$$(9)\quad \frac{[Cov(\hat{y}_{Area},\hat{y}_a)]^2}{[Var(\hat{y}_a)+Var(\hat{y}_\ell)]} \leq Cov(\hat{y}_{Area},\underline{\hat{y}}_a)'[\Sigma_\ell+\Sigma_a]^{-1}\ Cov(\hat{y}_{Area},\underline{\hat{y}}_a).$$

A change of notation for Hartley's estimator will make this proof clear.

$$\begin{aligned} Cov(\hat{y}_{Area},\hat{y}_a) &= Cov(\hat{y}_{Area},\hat{y}_{a(1)}+\hat{y}_{a(2)}+\ldots+\hat{y}_{a(k)}) \\ &= Cov(\hat{y}_{Area},\hat{y}_{a(1)})+Cov(\hat{y}_{Area},\hat{y}_{a(2)}) \\ &\quad + \ldots + Cov(\hat{y}_{Area},\hat{y}_{a(k)}) \end{aligned}$$

$$= Cov(\hat{y}_{Area}, \underline{\hat{y}}_a)'\underline{1}.$$

and,

$$Var(\hat{y}_a) + Var(\hat{y}_\ell) = \sum_{h=1}^{k} \sum_{h'=1}^{k} Cov(\hat{y}_{a(h)}, \hat{y}_{a(h')})$$

$$+ \sum_{h=1}^{k} Var(\hat{y}_{\ell(h)})$$

$$= \underline{1}' \Sigma_a \underline{1} + \underline{1}' \Sigma_\ell \underline{1}$$

$$= \underline{1}' (\Sigma_a + \Sigma_\ell) \underline{1}.$$

Thus, proving equation (9) resolves into proving:

$$(10) \quad \frac{[Cov(\hat{y}_{Area}, \underline{\hat{y}}_a)' \underline{1}]^2}{\underline{1}' (\Sigma_a + \Sigma_\ell) \underline{1}} \leq Cov(\hat{y}_{Area}, \underline{\hat{y}}_a)' (\Sigma_a + \Sigma_\ell)^{-1} Cov(\hat{y}_{Area}, \underline{\hat{y}}_a).$$

A theorem from matrix theory (Rao, page 60) says that if A is a positive definite mxm matrix, and $\underline{u}$ and $\underline{x}$ are m-vectors, then:

$$(11) \quad \frac{(\underline{u}'\underline{x})^2}{\underline{x}'A\underline{x}} \leq \underline{u}' A^{-1} \underline{u}.$$

This theorem is a result of the Cauchy-Schwartz inequality. Substituting

$$A = \Sigma_a + \Sigma_\ell,$$

$$\underline{u} = Cov(\hat{y}_{Area}, \underline{\hat{y}}_a)$$

$$\underline{x} = \underline{1},$$

into inequality (11) gives

$$\frac{[Cov(\hat{y}_{Area}, \underline{\hat{y}}_a)' \underline{1}]^2}{\underline{1}'[\Sigma_a + \Sigma_\ell] \underline{1}} \leq Cov(\hat{y}_{Area}, \underline{\hat{y}}_a)' (\Sigma_a + \Sigma_\ell)^{-1} Cov(\hat{y}_{Area}, \underline{\hat{y}}_a).$$

which is exactly inequality (10) completing the proof that:

$$Var(\hat{y}_{Strata}) \leq Var(\hat{y}_H).$$

5. EMPIRICAL STUDY

To illustrate how the four estimators $\hat{y}_{Area}$, $\hat{y}_{Screen}$, $\hat{y}_H$, and $\hat{y}_{Strata}$ compare, data for cattle and hogs from June 1974 area and list frame surveys conducted by the Statistical Reporting Service, U.S.D.A., in a midwestern state, were used to obtain totals and variances for each estimator. The estimates and standard errors from each frame estimating the overlap domain strata are presented in Table 1. These estimates are substituted into the $\underline{\hat{y}}_\ell$ and $\underline{\hat{y}}_{o\ell}$ vectors for the multiple frame estimator $\hat{y}_{Strata}$. Estimates and standard errors vary considerably between frames for the same stratum. Standard errors are much smaller for the list frame estimates in the larger strata than for the area frame estimates.

The variances of $\hat{y}_H$ and $\hat{y}_{Strata}$ are determined by $Var(\hat{y}_{Area}) - p_{opt}\,[Cov(\hat{y}_{Area}, \hat{y}_a)]$ and $Var(\hat{y}_{Area}) - \underline{p}'_{opt}\,[Cov(\hat{y}_{Area}, \underline{\hat{y}}_a)]$ respectively. The p_{opt} values, correlations between segment totals and overlap domain, and the amounts by which $Var(\hat{y}_{Area})$ is reduced to equal $Var(\hat{y}_H)$ and $Var(\hat{y}_{Strata})$ are presented for the estimators $\hat{y}_{Strata}$ and $\hat{y}_H$ in Table 2. Values of p_{opt} differ considerably between strata for the $\hat{y}_S$ estimator and from the p_{opt} obtained for $\hat{y}_H$. The value of $Cov(\hat{y}_{Area}, \hat{y}_a)$ summed over the strata is the same as for $\hat{y}_H$. Therefore, the p_{opt} values optimized on a stratum-by-stratum basis are weighted by the $Cov(\hat{y}_{Area}, \hat{y}_a)$ values for each stratum. The larger the weighted value of p_{opt} for $\hat{y}_{Strata}$ compared to the unweighted p_{opt} of $\hat{y}_H$ the smaller the variance of $\hat{y}_{Strata}$ relative to $Var(\hat{y}_H)$.

TABLE 1--List and Area Livestock Inventory Estimates by Livestock Stratum, June 1974

Multiple Frame Strata	Overlap Domain: List ($\hat{y}_{\ell}$) Stratum Estimates	List Standard Error	List Coefficient of Variation	Area ($\hat{y}_a$) Stratum Estimates	Area Standard Error	Area Coefficient of Variation
	(000)	(000)	(%)	(000)	(000)	(%)
Hogs & Pigs						
1 (Unknown)	44.0	8.5	19.3	44.7	22.8	51.1
2 (0-9 Hogs)	305.4	70.0	22.9	200.3	50.5	25.2
3 (10-49 Hogs)	140.1	26.9	19.2	195.1	76.6	40.8
4 (50-499 Hogs)	314.4	24.3	7.7	328.9	110.8	33.7
Cattle & Calves						
1 (Unknown)	363.6	56.5	15.5	175.8	33.2	18.9
2 (0-9 Cattle)	436.3	69.4	15.9	327.3	60.2	18.4
3 (10-49 Cattle)	1179.8	53.1	4.5	975.2	111.1	11.4
4 (50-499 Cattle)	1334.8	56.7	4.2	1263.7	180.9	14.3

TABLE 2--Multiple Frame Components for Estimates $\hat{y}_H$ and $\hat{y}_S$, June 1974

Estimator	Stratum 1/	p_{opt} 2/	$Corr(\hat{y}_{Area}, \hat{y}_a)$ 3/	$p_{opt}[Cov(\hat{y}_{Area}, \hat{y}_a)]$ 4/	Percent of $Var(\hat{y}_{Area})$ 5/
				(000,000)	(%)
	1	1.216	.187	918	3
	2	.324	.300	862	3
Hogs	3	.989	.583	7,806	25
	4	.995	.689	13,415	42
$\hat{y}_S$	Total	.927 6/	-	23,001	73
$\hat{y}_H$	Overall	.829 7/	.913	20,556	65
	1	.288	.152	337	1
	2	.406	.201	1,139	2
Cattle	3	.806	.388	8,034	15
	4	.915	.719	27,517	51
$\hat{y}_S$	Total	.841 6/	-	37,027	69
$\hat{y}_H$	Overall	.774 7/	.920	34,077	64

1/ List frame strata based on number of head in the operation.
2/ Weight attached to the list frame estimate.
3/ Correlation between area frame total and area frame overlap domain.
4/ Amount by which $Var(\hat{y}_{Area})$ is reduced to equal $Var(\hat{y}_{Strata})$ and $Var(\hat{y}_H)$.
5/ p_{opt} $[Cov(\hat{y}_{Area}, \hat{y}_a)]$ as percent of $Var(\hat{y}_{Area})$.
6/ Weighted value of p_{opt} for $\hat{y}_{Strata}$ from weighing individual stratum p_{opt} by stratum $Cov(\hat{y}_{Area}, \hat{y}_a)$.
7/ Value of p_{opt} for $\hat{y}_H$ ignoring strata.

Individual stratum p_{opt} values greater than 1 in effect give a negative weight to the area frame as in Stratum 1 for hogs and pigs. The size of p_{opt} depends upon $Var(\hat{y}_{ol})+Cov(\hat{y}_{nol},\hat{y}_{ol})$ relative to $Var(\hat{y}_{l})+Var(\hat{y}_{ol})$. If the covariance between the nonoverlap domain and the overlap domain for a given stratum is larger than the variance of the list estimate for that stratum then p_{opt} is greater than one. It is also noteworthy that for both hogs and cattle the contribution of the small livestock strata in reducing the total variance is minimal. This was due to smaller overlap domain standard errors for these strata relative to the higher strata and poor correlation between $\hat{y}_{Area}$ and $\hat{y}_{a}$ in the lower strata.

Except for Stratum 1 Hogs, the weight for the list frame, p_{opt}, and the correlation, $Corr(\hat{y}_{Area},\hat{y}_{a})$, were lower for the smaller strata. Even when the list frame weight was greater than one, the reduction in variance was small. The stratum-by-stratum multiple frame procedure provides a means of measuring the contribution of the frames for each stratum.

The combinations of estimates from each of the two frames into the various multiple frame estimates are presented in Table 3. There is little difference between results for the screening estimator ($\hat{y}_{Screen}$) and the Hartley estimator ($\hat{y}_{H}$). The weight attached to the list frame is so dominant that very little reduction in sampling error is realized from the contribution of the area frame overlap domain.

TABLE 3--Multiple Frame Livestock Estimates Using Alternative Estimators, June 1974

Multiple Frame Estimator	Hogs Estimate	Hogs Standard Error	Hogs Coefficient of Variation	Cattle Estimate	Cattle Standard Error	Cattle Coefficient of Variation
	(000)	(000)	(%)	(000)	(000)	(%)
$\hat{y}_{Area}$	1301.1	177.4	13.6	3615.4	231.4	6.4
$\hat{y}_{Screen}$	1299.9	106.6	8.4	4188.9	149.6	3.6
$\hat{y}_{H}$	1300.1	104.4	8.0	4067.1	139.5	3.4
$\hat{y}_{Strata}$	1265.4	92.0	7.3	3952.2	128.6	3.3

The $\hat{y}_{Strata}$ estimate for hogs was slightly below the other multiple frame estimates while the $\hat{y}_{Strata}$ cattle estimate was between $\hat{y}_{Area}$ and $\hat{y}_{Screen}$, and near $\hat{y}_{H}$. This reflects the relative size of the area frame estimate and its weight compared to the list frame estimate and its weight in each stratum.

The stratum-by-stratum combination of area and list frame estimates resulted in the smallest sampling errors for $\hat{y}_{Strata}$ as expected. Sampling errors on hog and cattle estimates were about 14% lower for $\hat{y}_{Strata}$ than the screening estimator, $\hat{y}_{Screen}$, and approximately 12% and 8% respectively below $\hat{y}_{H}$.

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